Quantifying Snow Volume Uncertainty from Repeat Terrestrial Laser Scanning Observations

Preston J Hartzell[1] (pjhartzell@uh.edu), Peter J Gadomski[1,2] (pjgadomski@uh.edu),
David C Finnegan[2] (david.finnegan@usace.army.mil), Craig L Glennie[1] (clglennie@uh.edu), Jeffrey S Deems[3] (deems@nsidc.org)

1. Motivation

- Terrestrial laser scanning (TLS) has become a common tool for 3D measurements of snow surfaces with subsequent snow depth and volume computations supporting applications such as avalanche prediction and snow hydrology (Deems et al. 2013). Although a number of studies have compared TLS-derived snow depths to independent *in situ* depths (e.g., Prokop 2008), none have rigorously quantified TLS point error as a function of the raw TLS observations, point cloud registration parameters, and observed terrain morphology.
- The goal of this work is to rigorously quantify expected errors in TLS points and examine their influence on derived differential snow volume products. The results are also expected to support decisions regarding TLS as a quality control mechanism for airborne laser scanning measurements of snow surfaces.

2. Test Datasets

Mammoth Mountain

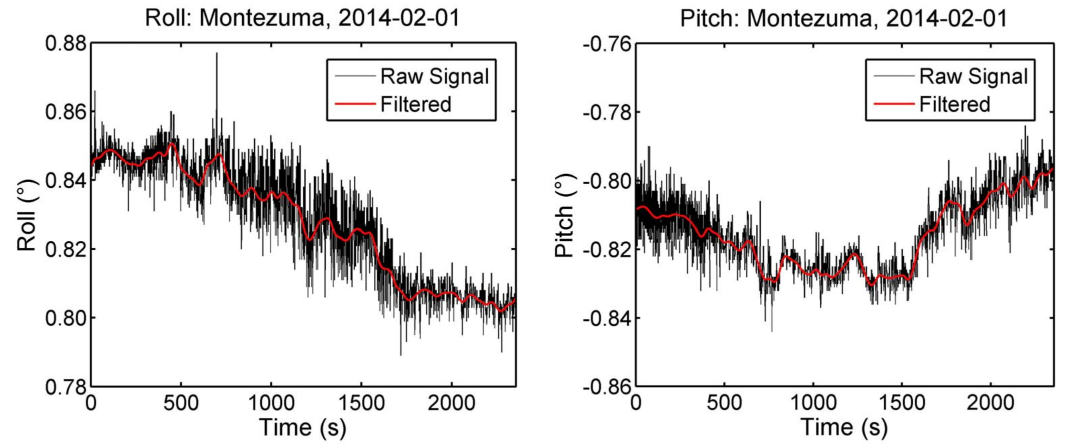
- CRREL, University of California Santa Barbara, Eastern Sierra Snow Study Site, CA
- Permanently mounted Riegl LMS-Z390i TLS (1.55 μm laser) observations of an ~560 m^2 study area every 15 minutes
- June and November 2012 datasets

Montezuma Bowl

- Arapahoe Basin Ski Area, Dillon, CO
- Tripod mounted Riegl VZ-4000 TLS (1.55 μm laser) observations of an ~150,000 m^2 study area
- June 2013 and February 2014 datasets

3. Data Preparation

1. Instrument movement correction (Montezuma snow-on scan only):
   - Instrument settling occurred due to tripod setup on snowpack, producing irregular vertical errors in excess of 60 cm at the study area extremities.
   - 1-second interval inclination sensor readings were extracted from the raw sensor file, low-pass filtered and applied as corrections to the observations.
   - Observed errors improved from 60+ cm to ~17 cm.

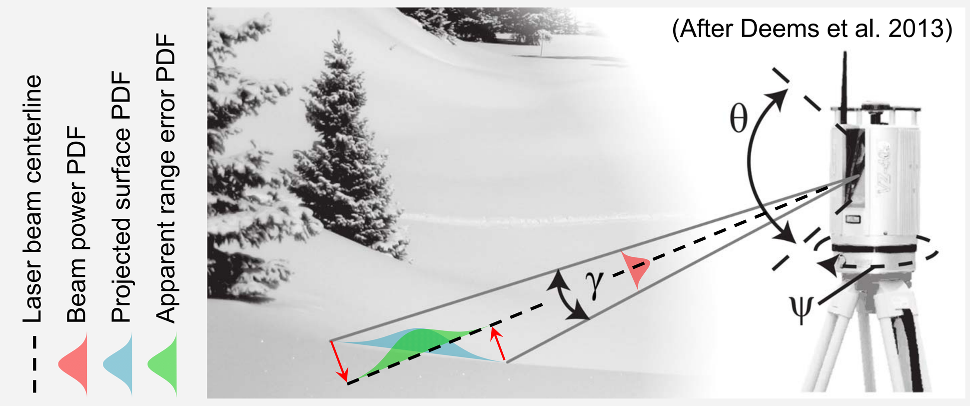


2. Point cloud registration via common planar and point features (Montezuma snow-on scan only):
   - Snow-off and snow-on datasets initialized with a coarse registration
   - Common features were selected and a custom least squares adjustment employed to solve for fine registration parameters.

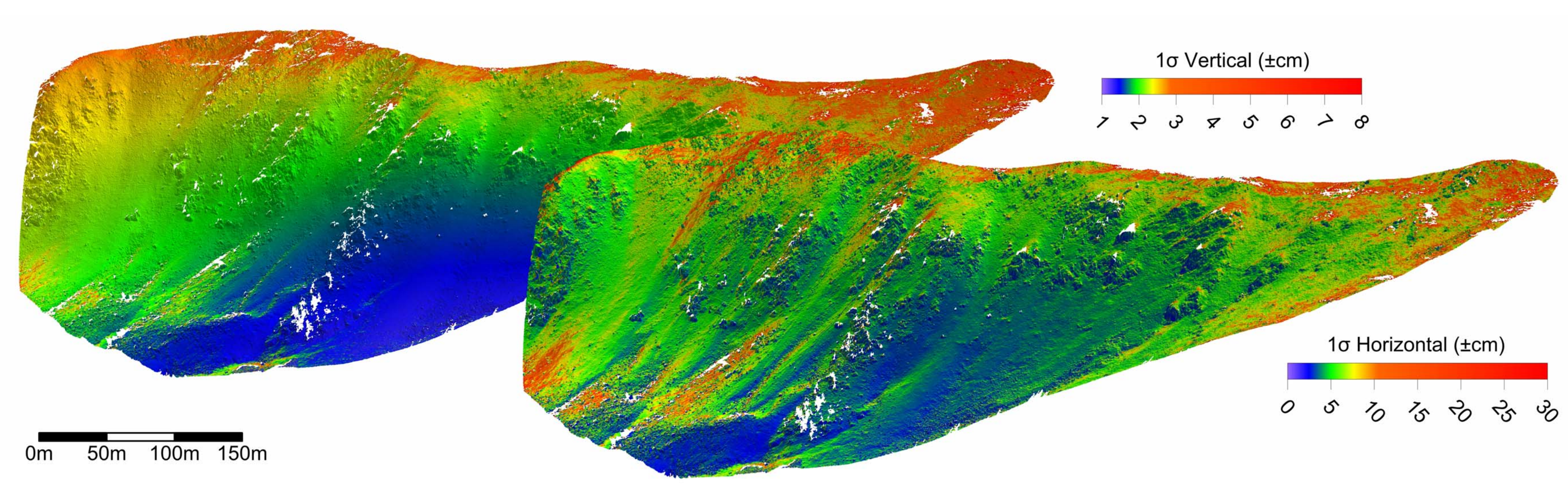
4. TLS Point Error Propagation

Box 1: Error Sources & Estimation

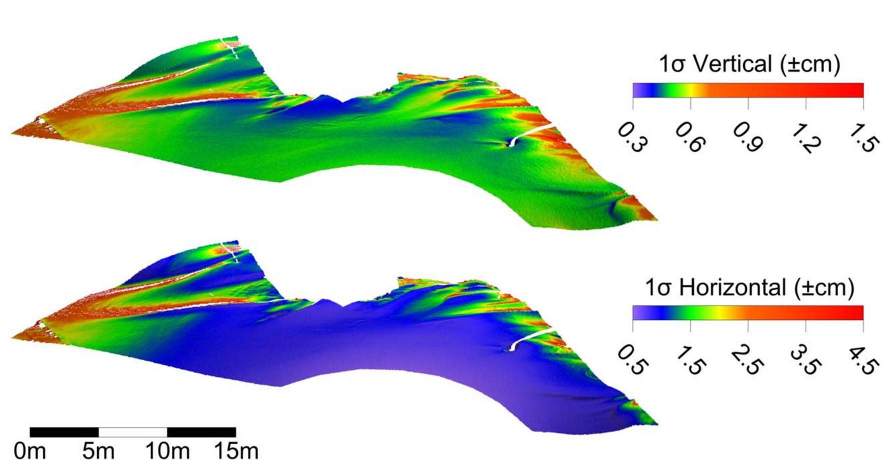
- TLS range (ρ), and hz. & vt. angle (ψ, θ) observation uncertainty
  - ⇒ Manufacturer specification sheet
- Cloud registration translation (T_x, T_y, T_z) and rotation (ω, φ, κ) parameter uncertainty
  - ⇒ Inverse normal matrix from least squares registration adjustment
- TLS inclination (*roll*, *pitch*) observation uncertainty
  - ⇒ Estimated from filtered inclination signal residuals
- Laser beam width uncertainty
  - ⇒ Beam exit diameter and divergence (γ) from specification sheet
  - ⇒ Following Lichti and Gordon (2004), the laser pulse power profile is interpreted as a probability density function (PDF) of the reflecting target location and applied to angular uncertainty.
  - ⇒ We transform the local planar projection of the PDF proposed in Schaer et al. (2007) to an equivalent range uncertainty along the beam centerline to estimate apparent range error as a function of beam width and local terrain morphology.



Montezuma: Snow-off Propagated Point Uncertainty



Mammoth: Snow-on Propagated Point Uncertainty



Box 2: Error Propagation

- Model:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_R = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}_F + \boldsymbol{R}_F \left[\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}_C + \boldsymbol{R}_C \boldsymbol{R}_I \begin{bmatrix} \rho\cos\theta\cos\psi \\ \rho\cos\theta\sin\psi \\ \rho\sin\theta \end{bmatrix}_S \right]$$

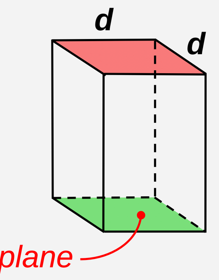
  - ⇒ $\boldsymbol{R}$ = 3×3 rotation matrix
  - ⇒ R = registered; F = fine; C = coarse; I = inclin.; S = scanner
- Propagation of Variance: $\boldsymbol{C}_{xyz} = \boldsymbol{A}\boldsymbol{C}_{ll}\boldsymbol{A}^T$
  - ⇒ $\boldsymbol{C}_{xyz}$ = 3×3 covariance matrix of a registered coordinate
  - ⇒ $\boldsymbol{A}$ = Matrix of partial derivatives (nonlinear model)
  - ⇒ $\boldsymbol{C}_{ll}$ = Covariance matrix of estimated model parameter errors

5. Volume Error Propagation

Box 3: *Gross Volume* Under a Raster Surface

$$v_r = d^2 \frac{\sum_{i=1}^{n} z_i}{n}$$

- Volume model based on rectangular prisms formed by vertically projecting each raster cell to the xy-plane
  - ⇒ n = #points in cell; d = raster dimension
  - ⇒ Total volume is the sum of all prisms
  - ⇒ Simple point binning means each point only influences one prism volume
- Standard propagation of variance applied to the model
  - ⇒ Only vertical point error influences the volume uncertainty

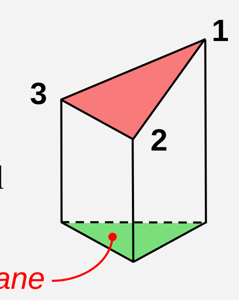


Box 4: *Gross Volume* Under a Mesh Surface

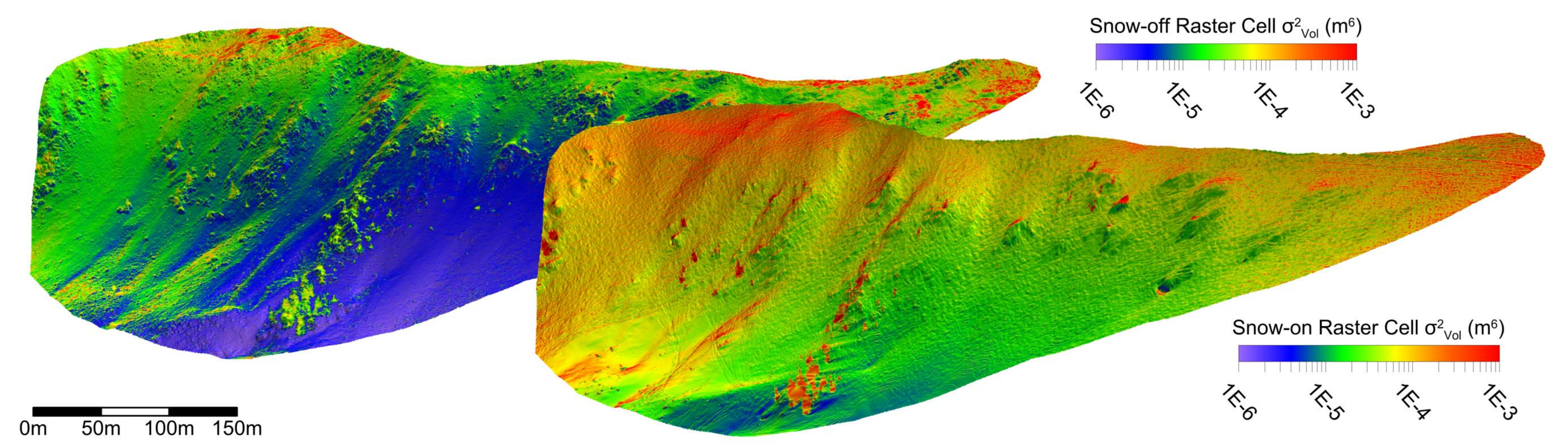
- Volume model based on truncated right triangular prisms formed by vertically projecting each mesh triangle to the xy-plane

$$v_m = \frac{1}{6}\left[x_1(y_2 - y_3) + x_2(y_3 - y_4) + x_3(y_1 - y_2)\right](z_1 + z_2 + z_3)$$

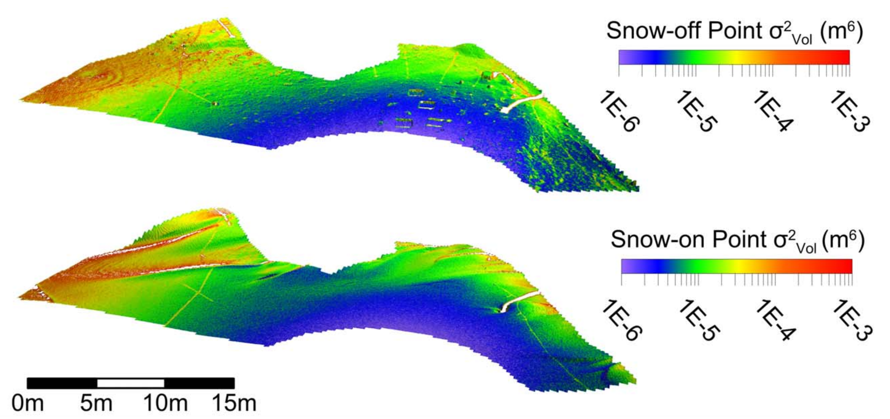
  - ⇒ Total volume is the sum of all prisms
  - ⇒ In a triangulated mesh, each point influences multiple prism volumes
- Standard propagation of variance applied to the model
  - ⇒ Both horizontal and vertical point error influence volume uncertainty



Montezuma: Snow-off & Snow-on Raster Cell Contribution to Volume Variance

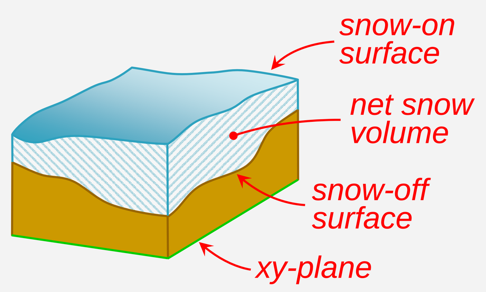


Mammoth: Snow-off & Snow-on Mesh Point Contribution to Volume Variance



Box 5: Typical *Net Snow Volumes* & 1σ Uncertainties

- Net snow volume is computed as the difference between the gross volume under the snow-on surface and the gross volume under the snow-off surface, both with respect to the common xy-plane datum.

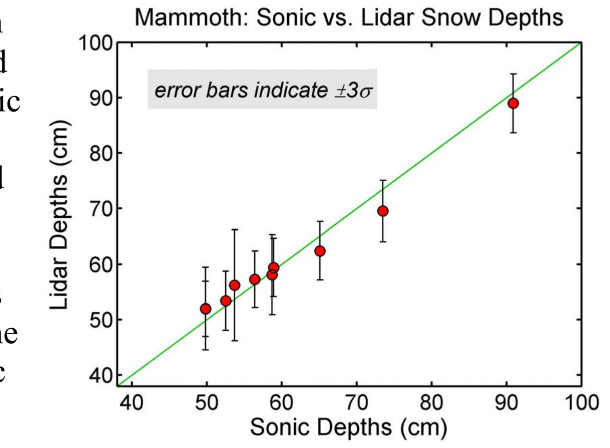


- Montezuma: 2013-09-14 subtracted from 2014-02-01
  - ⇒ Raster: 130,583.04 ± 4.743 m^3
  - ⇒ Mesh: 130,743.10 ± 10.541 m^3
- Mammoth: 2012-06-16 subtracted from 2012-11-11
  - ⇒ Raster: 395.767 ± 0.0182 m^3
  - ⇒ Mesh: 395.761 ± 0.0344 m^3

6. Results

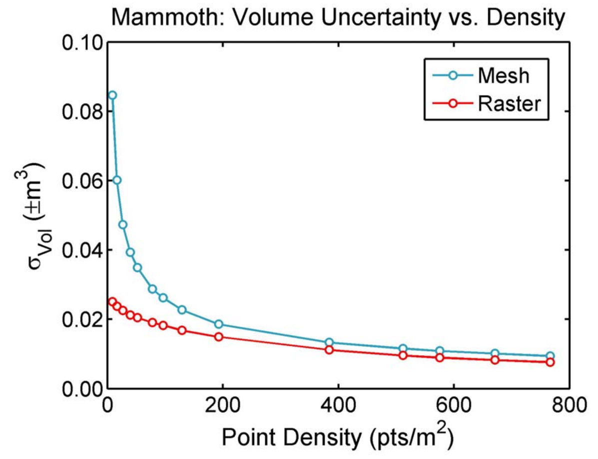
Snow Depth: TLS versus Sonic Depth Sensor

- Snow depths were estimated from 10 Mammoth scans and compared to measurements from a Judd sonic depth sensor. After removal of a mean bias of 9.7 cm, all sonic and lidar depths agree within their combined error estimates at 3σ. Potential causes for the mean bias include vegetation influence on the sonic bare ground range and sonic penetration of low density snow.



Point Density versus Estimated Volume Error

- Point density was systematically decreased in a June 2012 Mammoth scan and compared to computed volume uncertainty. Densities greater than 100 pts/m^2 provide minimal improvement in volume uncertainty.
- Mesh volume uncertainty is always greater than raster volume uncertainty due to the inclusion of horizontal point error.



7. Conclusions

1. After removal of a mean bias, snow depths derived from TLS points agree with sonic depth readings within the estimated error of the two measurement techniques. This suggests the TLS point positions and propagated coordinate uncertainties are reasonable.
2. The greater surface morphology fidelity captured by a mesh surface as compared to a raster surface is complemented by more rigorous volume error propagation that incorporates both horizontal and vertical 3D point errors. This is reflected in the larger volume uncertainties realized by mesh-based volumes than raster volumes.
3. Volume uncertainties are negligible compared to net snow volumes, and are surpassed by the volume difference between the raster and mesh methods for the Montezuma dataset. The surface generation method may therefore be more important than TLS observation accuracy in volume computation.
4. In light of the asymptotic improvement in volume accuracy with increasing point measurements, TLS observation densities greater than 100 pts/m^2 may only be advantageous for volume computations if attempting to capture surface morphology at scales smaller than 10 cm .

Acknowledgements

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[1]University of Houston, Houston, TX • [2]U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, NH • [3]National Snow and Ice Data Center, University of Colorado, Boulder, CO